Dynamics of Vapor Bubbles and Boiling Heat Transfer

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Analytical expressions for bubble radii and growth rates derived by the authors are applied in an analysis of surface boiling at high heat transfer rates. It is shown that the product of bubble radius and radial velocity is a constant, independent of the bubble radius. This circumstance permits the formulation of a Reynolds number for the flow in the thin superheated liquid layer adjacent to the heating surface. The result of the analysis is then applied to maximal heat transfer rates in pool boiling.

Modern technological developments in the fields of rocket engines and nuclear reactors, with their high heat transfer rates, have created renewed interest in the field of boiling heat transfer.

Most investigations are concerned with experiments on heat transfer rates and burnout conditions for pool boiling, forced convection with various degrees of subcooling, and natural-circulation evaporation.

It is generally agreed that the high heat transfer rates encountered with nucleate boiling are not a consequence of the latent heat transport but are due to the turbulence in the superheated liquid boundary created by bubble dynamics. In order to obtain a quantitative understanding of nucleate boiling, it is therefore necessary to take into account the mechanism of bubble formation and growth.

The authors have developed a theory for the growth of a vapor bubble in a superheated liquid (*1*) which is in good agreement with experimental data (*2*). The analysis has also been extended to volume boiling (*3*). In the present paper the results of the analysis are used in a quantitative formulation of the microconvection in surface boiling. To facilitate the presentation, experimental results of nucleate boiling are discussed and the theory of bubble growth is briefly reviewed.

NUCLEATE BOILING

Developments in nuclear reactors and rocket engines, where exceedingly high heat quantities are generated in comparatively small volumes, focused attention on nucleate boiling as a mode of transferring heat at high rates at an almost constant temperature of the heat transfer surface.

Jakob (*4*) proposed that this increase in heat transfer in nucleate boiling was due to agitation of the liquid near the wall caused by detaching bubbles. From a photographic study of nucleate boiling in subcooled water Gunther and Kreith (*5, 6*) showed the existence of a highly superheated (53°F.) film (0.005 in. in thickness) next to the heating surface. The high thermal resistance of this film is removed by the growth and collapse of vapor bubbles. Radial velocities of 8 to 15 ft./sec. were reported, indicating the importance of bubble agitation. From photographic bubble counts Gunther and Kreith quantitatively showed (*6*) that neither the latent heat transport by the growing bubbles nor a mechanism of simultaneous evaporation at the equatorial region and condensation at the bubble cap can account for the measured heat transfer rates. When they inserted the measured radial velocity and film thickness in the Sieder-Tate convective heat transfer equation, agreement with the experimental heat flux was obtained. The small contribution of the latent heat transport was also confirmed by Rohsenow and Clark (*7*).

In his studies on the mechanism of boiling heat transfer, Ellion (*8*) measured bubble radii and radial velocities in subcooled water and in carbon tetrachloride at atmospheric pressure. He used these measured values as the characteristic velocity and length in the Reynolds and Nusselt numbers and obtained good agreement with experimental heat fluxes. He also observed in his photographic studies that bubbles were ejected from the lower side of a horizontal heating strip—a motion which cannot be attributed to buoyant forces. At higher superheats the diameter of a bubble at departure is therefore governed by a different mechanism, which indicates the importance of the radial velocity of a bubble while it is still attached to the heating strip.

Corty and Foust (*9*) made photographic studies of nucleate boiling and demonstrated the importance of the size and distribution of the microroughness in determining the liquid superheat and boiling heat flux. A small increase of superheat in the nucleate region activates a very large number of nucleating centers, reflecting the extreme sensitivity of heat flux to nucleation. They also showed experimentally that the dimensions of the active surface cavities are closely represented by the critical radii as given by Gibbs's equation.

These experimental findings indicate that the large heat transfer rates associated with nucleate boiling are a consequence of the microconvection in the superheated sublayer. In order to arrive at a quantitative understanding of the process the dynamics of the vapor bubbles must be taken into account in formulating the analysis.

BUBBLE DYNAMICS

The first important work in bubble dynamics was done by Lord Rayleigh (*10*), who formulated it as a problem of the dynamics of an incompressible, inviscid fluid, obtaining the equation that now bears his name:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 + \frac{2\sigma}{\rho_L R} = \frac{P_v - P_\infty}{\rho_L} \tag{1}$$

As discussed by Frenkel (*11*), one condition for the breakdown of the liquid (boiling), is the existence of an embryonic bubble with critical radius defined by

$$R_o = \frac{2\sigma}{P_v - P_\infty} \tag{2}$$

The temperature at which the process of boiling can start must be higher than the saturation temperature T_∞ corresponding to the external pressure P_∞ (pressure on the system). In the case of an embryonic bubble with radius R_o this "starting" temperature T_o must exceed T_∞ by an amount corresponding to the excess value of P_v with respect to P_∞, i.e., by the capillary pressure $2\sigma/R_o$. The temperature of the bubble wall necessarily decreases owing to evaporation at the interface, whereby P_v on the right side of Equation (1) becomes time dependent. The temperature of the vapor within the bubble is practically the same as the temperature of the bubble wall. The determination of the latter temperature involves the solution of a problem of heat conduction in a moving medium with given motion of the boundaries. The analysis(12) furnishes the instantaneous temperature $T_v(t)$ inside the vapor bubble, which at time zero was situated in an infinite liquid of temperature T_o and which then increased in size owing to evaporation, attaining radius R (z) at time z during the interval $0 \leqslant z \leqslant t$

$$T_v(t) - T_o = -\frac{L\rho_v}{C\,\rho_L\sqrt{\pi a}} \int_o^t \frac{R(z)\,\overset{\circ}{R}(z)}{R(t)\sqrt{t-z}}\,dz \qquad (3)$$

It was pointed out by Forster (13) that a rather general class of problems of heat transfer in moving media with given motion of the boundaries may be solved by an extension of the foregoing method.

Combining Equations (1) and (3) and the Clausius-Clapeyron equation yields the integrodifferential equation which describes the growth of a spherical bubble in a superheated liquid:

$$R\overset{\circ\circ}{R} + \frac{3}{2}\overset{\circ}{R}^2 + \frac{2\sigma}{\rho_L R} = \frac{L}{\rho_L T(V_v - V_L)} \left[T_o - T_\infty - \frac{L\rho_v}{C\,\rho_L\sqrt{\pi a}} \int_o^t \frac{R(z)\,\overset{\circ}{R}(z)}{R(t)\sqrt{t-z}}\,dz\right] \qquad (4)$$

Based on mathematical arguments, the inertia of the liquid, represented by the terms ($R\overset{\circ\circ}{R} + 3/2\,\overset{\circ}{R}^2$), was shown to be of minor importance in determining bubble growth. Were this not the case, the complexity of the solution of Equation (4) would forbid the application of the results to bubble populations as encountered in boiling. Since this point is important for the present analysis, it is here further considered from the point of view of thermodynamics.

While the bubble of radius R expands by dR, heat energy δQ is taken from the surroundings and $\delta Q > L\rho_v 4\pi R^2 dR$; most of δQ is used to increase the internal energy of the system and to do work against the atmospheric pressure. The mechanical work δW_m done against combined surface tension and liquid inertia is bounded by $\delta W_m \leqslant \Delta P 4\pi R^2 dR$ because the pressure in the bubble is at most ΔP above atmospheric. The ratio $\delta W_m/\delta Q$ therefore remains smaller than $\Delta P/L\rho_v$ and the latter expression is equal to $\Delta T/T$ by Clausius-Clapeyron's equation. Hence, of the energy reaching the bubble, less than the fraction $\Delta T/T$ is at any time available for doing work against surface tension and liquid inertia combined. For boiling liquids this fraction is of the order of a few per cent.

We want to find a bound for the fraction of the energy available for doing work against the liquid inertia alone. During expansion by dR the work done against surface tension δW_s equals $8\pi\sigma R dR$ and with $2\sigma = R_o\Delta P$ this work equals $4\pi R R_o \Delta P dR$; thus $\delta W_s/\delta W_m$ equals R_o/R. The fraction η of δQ, which is available for accelerating the liquid, is therefore bounded by the inequality

$$\eta \leqslant \frac{\Delta T}{T}\left(1 - \frac{R_o}{R}\right) \qquad (5)$$

In the initial stage of growth, while the temperature of the bubble wall is still close to T_0, the radius R is also still close to the initial radius R_o and therefore the factor $(1 - R_o/R)$ in *Equation* (5) is then close to zero. Later, when R has grown sufficiently to make this factor of order unity the other factor $\Delta T/T$, which was small at the start, has further decreased and is approaching zero just as the bubble-wall temperature decreases and approaches T_x. The fraction η of the energy available for accelerating the liquid is thus seen to start at zero, to be always bounded by a theoretical bound of a few per cent, and to approach zero again. It may be concluded that the liquid inertia is not a controlling factor in the growth of a vapor bubble if the growth takes place by evaporation. Statements to the contrary which appeared in the literature without proof(14) are therefore in error.

Considerations similar to those here presented facilitate the mathematical analysis to such an extent that a solution of the problem in closed form can be obtained(1):

$$r + \ln\frac{r-1}{r_1 - 1} = C\sqrt{t}\,;\; r > r_1 \qquad t > \frac{r_1^2}{C^2} \qquad (6)$$

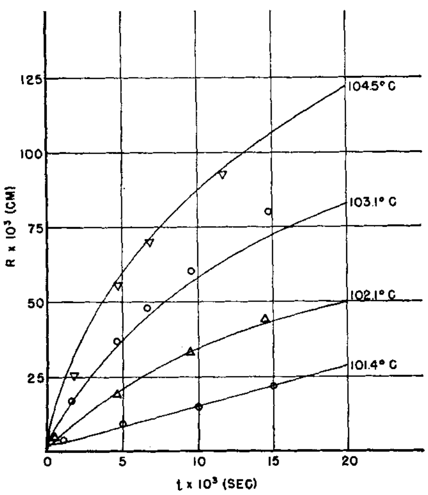
where

$$r = \frac{R}{R_o},\quad R_o = \frac{2\sigma}{\Delta P} \qquad (7)$$

The bubble-growth coefficient C is

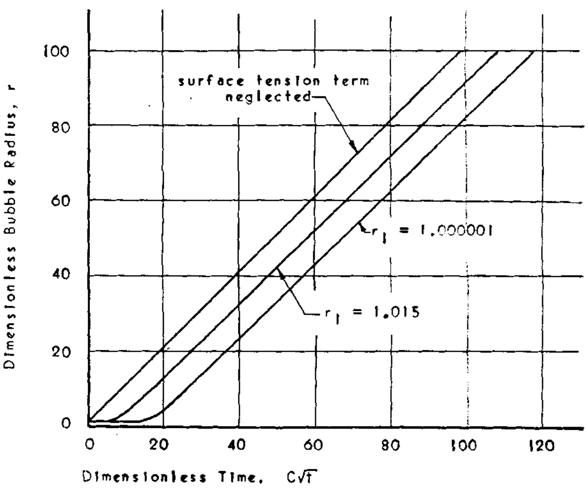
$$C = \frac{\Delta T c\,\rho_L\sqrt{\pi a}}{L\,\rho_v\,R_o} \qquad (8)$$

The radius r_1 is not arbitrary but calculable, and it turns out to be only 1 or 2% above the critical radius ($r = 1$). Bubble radii as predicted by Equation (6) and compared with experiment(2) are shown in Figure 1 for illustration (cf. reference 1).



← Fig. 1. Radius-time relation for vapor bubbles growing in superheated water as given by Equation (6). [Circles and triangles show experimental values by Dergarabedian (2).]

→ Fig. 2. Dimensionless bubble radius as function of dimensionless time, from Equation (6).



The logarithmic term in Equation (6) stems from the surface-tension term in Equation (4). It may be seen from its logarithmic nature that it is most important only while the bubble radius has increased by less than an order of magnitude. Afterward its influence consists mainly in a shift along the time axis, leaving practically unaffected the growth rate of bubbles considerably larger than the critical. This fact is illustrated in Figure 2.

It was shown (*15*) that to the extent that the inertia term is unimportant the bubble-wall temperature T_v is given by

$$T_o - T_v = (T_o - T_\infty)\left(1 - \frac{1}{r}\right) \quad (9)$$

Hence the vapor temperature approaches the saturation temperature T_∞ as soon as r has grown by an order of magnitude.

In the foregoing considerations the relative importance of the various factors influencing bubble growth was evaluated and compared. It was demonstrated that the growth of a vapor bubble in a superheated liquid [Equation (6)] is essentially determined by the single coefficient C given by Equation (8).

The question now arises whether the former analysis can be applied to the nonspherically symmetric bubble next to the heating surface. The following comparison of measured and predicted initial radial velocities of bubbles still attached to the heating surface is therefore significant.

Ellion (*8*) reported a typical initial growth rate of 10 ft./sec. for a bubble-radius increase from 0.001 to 0.010 in. in degassed, subcooled water at a temperature of 135°F. and 74% of peak (burnout) flux. The peak flux for this temperature is 3 B.t.u./(sq.in.)(sec.); hence for 2.22 B.t.u./(sq.in.)(sec.) the corresponding excess wall temperature (maximum superheat) is 48°F. (cf. reference *8*, Figures 38 and 39).

It was previously shown that the influence of the surface-tension term on the rate of growth is small in the domain here considered. Therefore it follows from Equations (6) and (8) that the radial velocity is given with good approximation by

$$\overset{\circ}{R} = \frac{\Delta T c\, \rho_L \sqrt{\pi a}}{L \rho_v} \frac{1}{2\sqrt{t}} \quad (10)$$

If the properties of the liquid at the superheat temperature and those of the vapor at the saturation temperature are evaluated, the coefficient $\Delta T c \rho_L \sqrt{\pi a}/L\rho_v$ for a superheat of 48°F. is 4.54 cm./$\sqrt{}$sec.; the time period in Ellion's experiment is 75×10^{-6} sec. Equation (10) then predicts a velocity of 11.2 ft./sec., which is to be compared with the measured initial growth rate of 10 ft./sec. The predicted velocity could have been expected to be larger than the average experimentally measured velocity, since the computation was based on the maximum superheat. The bubble actually grows through a film region where a high-temperature gradient exists, and so in reality it experiences a somewhat lower mean superheat. With a mean superheat of, say, 38°F. the predicted initial growth rate would be 8.75 ft./sec. Similar agreement is obtained with experimental data reported in references 5 and 6. It may thus be seen that the theory compares favorably with experiment.

It should be noted that the coefficient $\Delta T c \rho_L \sqrt{\pi a}/L\rho_v$ decreases rapidly with an increase in pressure; thus the agitation from one nucleating center is greatly reduced at elevated pressure. Similarly, if the Laplace constant $[2\sigma/g(\rho_L - \rho_v)]^{\frac{1}{2}}$ be a measure of the agitation introduced by a departing bubble from *one* nucleating center, its agitation effect per unit time would again decrease with pressure because the Laplace constant changes by less than an order of magnitude and the growth rate decreases in the same pressure range by several orders of magnitude. Both facts indicate that the higher heat flux at higher pressures must be due to an increase in the number of nucleating centers as pressure is increased.

One may see the physical reason for this increase in the number of nucleating centers with an increase in pressure by considering how (for a given roughness distribution and temperature difference) the nucleation propensity depends on the surface tension (which decreases) and the slope of the vapor pressure-temperature curve (which increases).

MICROCONVECTION AND BOILING HEAT TRANSFER

Application of the theory of similarity to the differential equations of heat transfer and fluid flow (*16, 17*) yields the result that for the heat transfer from a solid boundary to a fluid, in similar systems, a relation

$$Nu = \phi\,(Re, Pr) \quad (11)$$

will exist. The function ϕ may be formulated from experimental data, and for purposes of design, an empirical expression of the form

$$Nu = \alpha\, Re^m Pr^n \quad (12)$$

is usually employed.

It is surmised that an expression of the form of Equation 12 will describe the process of heat transfer in boiling; the problem is to formulate the Reynolds and Nusselt numbers in terms of those parameters which are most descriptive of the essential features of the physical system.

For the Reynolds number a characteristic length and velocity must be found. Various authors (*18, 19*) chose for their characteristic length the diameter of the bubble at departure from the heating surface, which is specified by the Laplace constant and the contact angle; this diameter multiplied by the frequency of bubbles leaving the surface was used as a characteristic velocity. The physical significance of the length thereby obtained consists in its relation to the largest size of bubble for which buoyant and adhesive forces are still in equilibrium. Although the equilibrium of these static forces is certainly physically significant at low superheats and low levels of agitation, it must be expected that for high superheat and high levels of agitation the dynamic forces due to fluid motion will be more important. In this connection recent studies by Yamagata (*20, 21*) and coworkers are of interest because they show the mutual interaction of bubbles at high heat fluxes. The characteristic velocity, arrived at as mentioned above, is of the order of 1,000 ft./hr. while the experimental and analytical evidence, as discussed previously, gives radial velocities of the order of 36,000 ft./hr.

Inasmuch as the state of liquid motion is most important in the thin layer of fluid adjacent to the heating surface, where also most of the temperature drop occurs, it is plausible that the bubble radii and radial velocities should furnish the characteristic length and velocity for the Reynolds number of the flow system. As mentioned be-

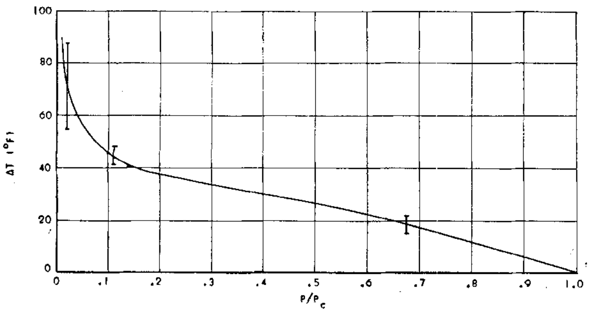


Fig. 3 Empirical correlation of maximum temperature difference (superheat) at peak flux in pool boiling by Cichelli and Bonilla(*22*).

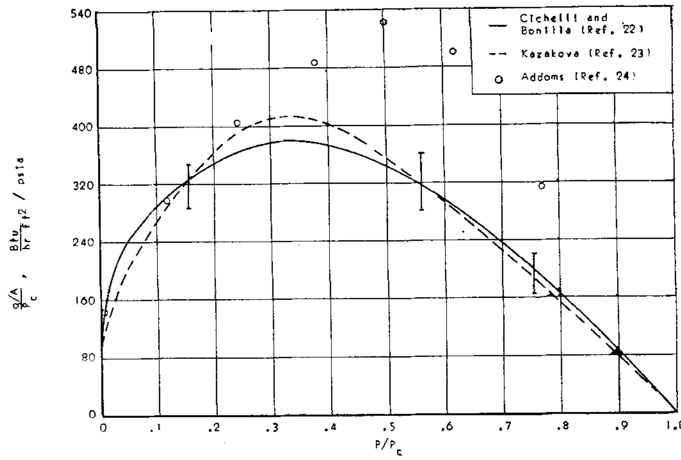


Fig. 4 Empirical correlation of maximum heat flux in pool boiling by Cichelli and Bonilla(*22*).

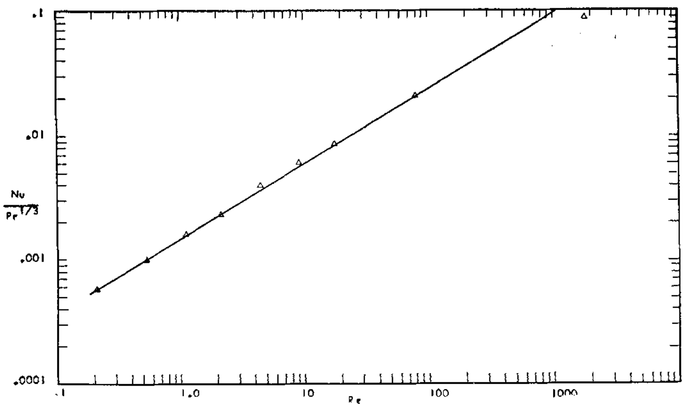


Fig. 5. Correlation of data for ethanol at maximum heat flux and temperature in pool boiling, from Equation (17).

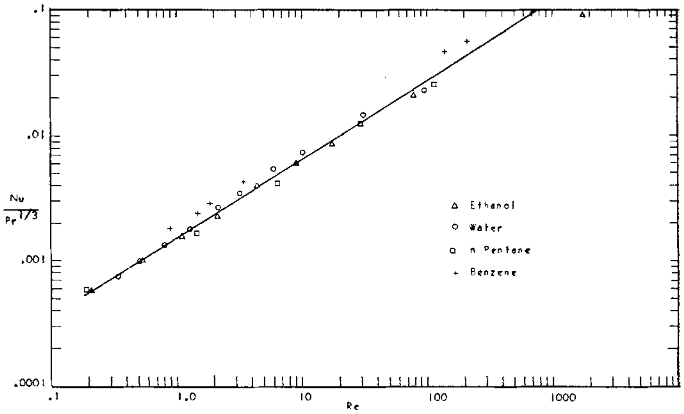


Fig. 6. Correlation of data for various liquids at maximum heat flux and temperature in pool boiling, from Equation (17).

fore, Ellion(*8*) in his Ph.D thesis with Professor Sabersky used such bubble radii and radial velocities, which he had to obtain from averages over many bubbles photographically observed in boiling water and carbon tetracloride at atmospheric pressure. The present theory gives mathematical expressions for the bubble radii and, significantly, the question of which bubble radius and which velocity should be chosen does not arise: As follows from the analysis given previously, the product

$$2R\mathring{R} = \left[\frac{\Delta T\, c\, \rho_L \sqrt{\pi a}}{L\rho_v}\right]^2 \quad (13)$$

is constant for a given superheat and pressure and is a function of the thermodynamic properties of the liquid and vapor only. Physically, the relation expressed by Equation (13) means that small bubbles grow faster and large bubbles grow slower in such a way that their contribution to the agitation of the fluid remains constant. The Reynolds number for the flow system (the superheated sublayer) is then

$$Re = \frac{\rho_L}{\mu}\left(\frac{\Delta T\, c\, \rho_L \sqrt{\pi a}}{L\rho_v}\right)^2 \quad (14)$$

The Nusselt number for the system is

$$Nu = \frac{q/a\, R}{(T_w - T_L)\, k} \quad (15)$$

where the length R is obtained from considerations of bubble dynamics and is given by

$$R = \frac{\Delta T\, c\, \rho_L \sqrt{\pi a}}{L\rho_v}\sqrt{\frac{2\sigma}{\Delta P}}\sqrt[4]{\frac{\rho_L}{\Delta P}} \quad (16)$$

The time constant represented by the roots in Equation (16) is not the only one that may be significant; other possibilities are at present under consideration.

Equations (12), (14), and (15) then yield a relation between the heat flux and the superheat in terms of the thermodynamic properties of the vapor and the liquid:

$$\frac{q/a}{(T_w - T_L)\, k}\left(\frac{\Delta T\, c\, \rho_L \sqrt{\pi a}}{L\rho_v}\sqrt{\frac{2\sigma}{\Delta P}}\sqrt[4]{\frac{\rho_L}{\Delta P}}\right) = \alpha\left[\frac{\rho_L}{\mu}\left(\frac{\Delta T c\, \rho_L \sqrt{\pi a}}{L\rho_v}\right)^2\right]^m\left[\frac{\mu c}{k}\right]^n \quad (17)$$

If the considerations which led to the formulation of Equation (12) are still valid for the process here considered, the exponent for the Reynolds number should be in the range $0.5 < m < 0.8$ and the exponent for the Prandtl number should be around 1/3.

COMPARISON WITH EXPERIMENTS

Many experiments on heat transfer in pool boiling are reported in the literature. Conditions of maximum heat flux (i.e., burnout conditions) are of great interest to the designer. Cichelli and Bonilla (*22*) have shown that the experimentally determined superheats of various organic liquids at burnout can be represented by a single curve giving the burnout superheat as a function of the reduced pressure (cf. Figure 3). They also showed that the burnout heat fluxes may similarly be correlated as a function of the reduced pressure (cf. Figure 4).

The validity of the theoretical considerations presented in the previous section can now be tested by investigating whether or not one equation [Equation (17)] with superheats inserted from Figure 3 yields heat-flux values in agreement with Figure 4.

Such calculations were first carried out for one liquid: ethanol. Superheats and corresponding heat fluxes were taken from the curves in Figures 3 and 4. Figure 5 shows the relation between the Nusselt, Reynolds, and Prandtl numbers as given by Equation (17) for a pressure range of 0.015 P_{crit} to 0.8 P_{crit}, that is, from 14.7 to 743 lb./sq.in.abs.* The linearity of the relationship indicates the constancy of the exponent m for which the value of 0.61 is found. A few words may be said about the point corresponding to atmospheric pressure, which deviates from the straight line. The superheat plotted in Figure 3 represents the difference between wall and saturation temperatures, i.e., the maximum superheat in the fluid. The bubble spends most of its time in surroundings below the maximum temperature, and, inasmuch as in the low-pressure region high superheats occur, errors in superheat values are most important in this region. A reduction of superheat by 10% would bring the point in question down to the straight line.

The results of the analysis as applied to n-penthane, benzene, ethanol, and water are presented in Figure 6. It may be seen that the various liquids follow closely the same relationship, given by

$$Nu = 0.0015\, Re^{0.62}\, Pr^{0.33} \quad (18)$$

where the dimensionless groups are given by Equation (17). Since Equation (17) is very sensitive to the superheat used, the foregoing correlation is in additional support of the analysis.

In the literature two sets of experimental data for water at burnout conditions at high pressure are reported, one by Kazakova (*23*) and one by Addoms, as reported by McAdams (*24*); both are shown in Figure 3. Kazakova's data are seen to follow closely the curve given by Cichelli and Bonilla. Kazakova also points out that the effects of rapid heat corrosion and salt deposition in increasing burnout heat transfer values become very pronounced at pressures above 64 atm. (0.3 P_{crit}). It is seen that the data reported by Addoms start to deviate from the data of Cichelli and Bonilla and those of Kazakova in just that pressure range. Inasmuch as the experimental conditions under which Addoms's data were obtained are not published, Kazakova's data for the pressure range 0.1 P_{crit} to 0.8 P_{crit} were used in the calculations for water represented on Figure 6.

The thermodynamic properties of liquids and vapors used in this paper were taken from the International Critical Tables and reference *22*. Liquid properties were evaluated at superheat temperatures; those of the vapor were taken at saturation temperature.

The considerations here presented, which compare favorably with experimental results, should prove helpful in he analysis of boiling of subcooled liquids. The bubble-growth coefficient, the importance of which in pool boiling was here demonstrated, should be equally significant for other conditions of boiling.

*In the graphs low Reynolds numbers correspond to high pressures.

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NOTATION

a = thermal diffusivity
$c_p = c$ = specific heat at constant pressure
g = acceleration due to gravity
k = thermal conductivity
L = latent heat of vaporization
P = pressure
$\Delta P = P_o - P_\infty$ = vapor - pressure difference corresponding to the superheat temperature
$r = R/R_o$ = dimensionless bubble radius
R = bubble radius
$R_o = 2\sigma/\Delta P$ = radius of critical bubble
t = time
T_w = temperature of the heating surface
T_o = temperature of the liquid
T_∞ = saturation temperature corresponding to pressure on system P_∞
$\varphi T = T_o - T_\infty$ = superheat temperature
v = specific volume
ρ = mass density
σ = surface tension
μ = viscosity

Subscripts

L = liquid
v = vapor

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